ISSN 0005-1179 (print), ISSN 1608-3032 (online), Automation and Remote Control, 2025, Vol. 86, No. 5, pp. 484–497.

AUTOMATION IN INDUSTRY

Determination of Optimal Parameters of a Model Predictive Controller with Priorities of Control Objectives under Conditions of Plant Uncertainty

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Received May 15, 2024

Revised June 19, 2024

Accepted July 25, 2024

Abstract—This work addresses the problem of determining the optimal parameters of a model predictive controller under conditions of plant parametric uncertainty, unmeasured disturbances, and the need to account for the prioritization of control objectives. An algorithm is proposed for determining the optimal parameters of a model predictive controller, which can be applied to both newly developed and existing control systems with predictive models, without requiring structural modifications. The algorithm incorporates a criterion that accounts for plant parametric uncertainty, the magnitude of output variable deviations beyond specified limits, and the duration of constraint violations. The effectiveness of the proposed algorithm is demonstrated through its application to an existing controller for a complex distillation column within a hydrocracking process unit. Using the algorithm, it was possible to achieve the minimum permissible initial boiling point of the kerosene fraction withdrawn from the bottom of the stripping column equipped with a thermosiphon, even under conditions of reduced coolant flow.

Keywords: model predictive control, parametric uncertainty, distillation column, hydrocraking unit

DOI: 10.31857/S0005117925050107

1. INTRODUCTION

In the oil refining industry, predictive model-based controllers are widely used due to their ability to control complex, multidimensional processes while operating under constraints on both control and controlled variables [1, 2]. A major advantage of these controllers is their capacity to optimize production performance, for example, by maximizing the yield of high-value products while ensuring compliance with quality specifications and minimizing energy consumption. Due to the multidimensional nature of process systems, predictive model-based controllers utilize weight matrices for input and output variables to achieve control objectives. However, these objectives can be compromised by plant parametric uncertainty, unmeasured external disturbances, or the use of suboptimal controller parameters [3].

Existing methods for ensuring the fulfillment of control goals can be divided into three groups. The first group includes methods that significantly change the common standard algorithms for model predictive controllers (MPCs). These methods are generally not suitable for existing systems, as they demand significant structural changes, which are often impractical in real-world applications. Thus, in the paper [4], the correct enforcement of priorities for controlled variables (CVs) is achieved by determining the optimal changes in manipulated variables (MVs) through a lexicographic optimization approach. This involves sequentially solving multiple optimization problems, where each subsequent problem includes additional constraints that preserve the solutions obtained in the previous steps. A similar approach is presented in paper [5], which also addresses the prioritization of CV by solving a sequence of optimization problems. The method of taking into account the priorities of CV is known, taking into account the restrictions on the state of the plant by solving the problems of mixed-integer programming with additional restrictions that penalize the violation of the priority of the control objectives [6]. These works address only the prioritization of CV and do not consider the effects of plant parametric uncertainty or unmeasured disturbances. The method of handling the parametric uncertainty of the plant involves online identification of controller model parameters using the recursive least squares method [7]. However, this method requires persistent excitation of the system in a closed-loop configuration, even in the presence of unknown and unmeasured external disturbances. It is also important to note that the methods described above generally entail a high computational burden, primarily due to the repeated execution of optimization routines.

The second group includes methods for online adjustment of weight matrices of controllers that do not require significant changes to existing algorithms and are a superstructure over existing systems. The paper [8] proposes a 2-stage method for taking into account priorities with the adjustment of weight matrices online. In the first stage, using the solution of the linear programming problem, the optimal achievable target values of CV are determined, taking into account the limitations and priorities of the control goals, as well as the optimal weight matrices of the controller. In the second stage, the controller with updated parameters issues optimal MV increments that provide tracking of target CV values. The method of adjusting weight matrices using a linear approximation of the relationship between the predictions of the output variables and the values of the weight matrices, as well as the subsequent solution of the linear programming problem with constraints, is known [9]. In this paper, the criterion of the optimization problem is based on the assessment of the quality of the transient process and does not take into account the uncertainty and priorities of CV. Paper [10] is devoted to the definition of weight matrices, taking into account the uncertainty of the plant in online mode by solving the optimization problem based on the simulation of the controller and aimed at minimizing the CV mismatch and minimizing the MV increment. Paper [11] considers the application of the particle swarm optimization (PSO) method for online adjustment of weight matrices of the controller without taking into account the uncertainty of the plant, control goals, and CV priorities; a root-mean-square error is used as a criterion for the optimization problem.

The third group includes methods for offline determination of weight matrices that do not require any changes in the parameters of the controller. Thus, in paper [12], using PSO, the weight matrices of the controller and the dimensions of the prediction and control horizons are determined, taking into account the uncertainty and priorities of CV in order to minimize CV mismatch. It should be noted that in this paper, in order to take into account the priorities of CV, it is necessary to set weights in the PSO criteria, which increase the penalty for violations for higher-priority variables. It is indicated that it is necessary to accurately select the weights in the criteria in order to obtain the expected result. However, in this case, the task of determining the weights that ensure the fulfillment of the priorities of CV is “transferred” from the weight matrices of the controller to the weights of the scalarized criterion. A similar method [13] for optimizing the weight matrices of the controller using PSO is known, taking into account the uncertainty, but this method does not take into account the priorities of CV. The paper [14] proposes a method for optimizing weight matrices using a genetic algorithm with a criterion based on fuzzy logic, which allows describing CV priorities and minimizing control error, without determining the weights related to CV in the genetic algorithm criterion. A method is known for determining weight matrices without taking into account the uncertainty and priorities of CV by solving a multi-criteria optimization problem. This approach seeks a compromise solution — known as a Pareto-optimal point — where improving one criterion is not possible without degrading the other. The criteria used are CV control error and MV increment [15]. In a similar work [16], also considering the task of determining the optimal weight matrices as multi-criteria, CV priorities are taken into account. This approach requires specifying time constant multipliers for the controller models used in the optimization criteria, ensuring that higher-priority CVs respond more quickly than lower-priority ones. However, it does not define the specific difference in the response rate needed to guarantee strict adherence to priority levels. In [17], the weight matrices of the controller are optimized in order to increase the speed of the transient process and minimize the CV error. In [17], several operating points are considered when determining the optimal weights, and the uncertainty of the plant and the priority of CV are not considered.

In contrast to the references considered above, this paper proposes an algorithm for finding the ratios of the weight matrices of the controller based on the predictive model, taking into account the control objectives (CV priority), the parametric uncertainty of the plant, and the unmeasured disturbances acting through the control channel. Also, the proposed algorithm can be used both at the stage of synthesis of new MPCs and to adjust the parameters of existing MPCs without the need to change the structure of the existing control algorithm. In the present paper, it is proposed to consider the ratios of the weight matrices, since the behavior of the MPCs is mainly determined by the ratios, and not by the absolute values of the weights [9]. The application of the proposed algorithm for determining the parameters of the existing MPCs of the complex distillation column of the hydrocracking process unit is considered. It is demonstrated that the developed algorithm for determining the optimal weight matrices of the MPC, based on the proposed criterion, enables the achievement of the minimum permissible initial boiling point of the kerosene fraction and ensures its maximum recovery from the bottom of the stripping column with a thermosiphon under conditions of low coolant flow. Additionally, the algorithm provides stable liquid accumulation at the bottom of the stripping column.

2. PLANT AND CONTROL SYSTEM DESCRIPTION

A complex distillation column of the hydrocracking process unit is considered, designed to separate the incoming stable hydrogenate into a gasoline fraction, a kerosene fraction, a middle distillate, a heavy diesel fraction, and a hydrocracking residue. Figure 1 shows a flowsheet diagram of the considered process unit.

One of the target products of this complex distillation column is the kerosene fraction. In this regard, the control task under consideration is to maximize the withdrawal of the kerosene fraction by involving lighter hydrocarbons of the gasoline fraction in it. This redistribution is achieved by minimizing the initial boiling point (FC IBP) of the kerosene fraction through three key actions: decreasing the top temperature of column K1, reducing the bottom temperature of column K2, and increasing the 98% boiling point (FC 98%) of the kerosene fraction by maximizing the flow rate of the column K2 output to the boundary limit. The bottom temperature of column K2 is regulated by adjusting the flow rate of coolant through the thermosiphon reboiler (TO2), which operates without a steam space. The coolant flow rate is controlled by valves FV113 and FV113B. It is noted that in the area of low coolant flow rates, i.e. when opening the valve FV113 less than 10%, stable accumulation of liquid in the bottom of column K2 can not be maintained. In such cases, the control mode is typically switched from automatic to manual by the operator to restore stability. Based on this operational constraint, the following control objectives are defined in descending order of priority: (1) ensure stable liquid accumulation in the bottom of column K2, (2) minimize the FC IBP of the kerosene fraction, and (3) maximize the recovery of the kerosene fraction.

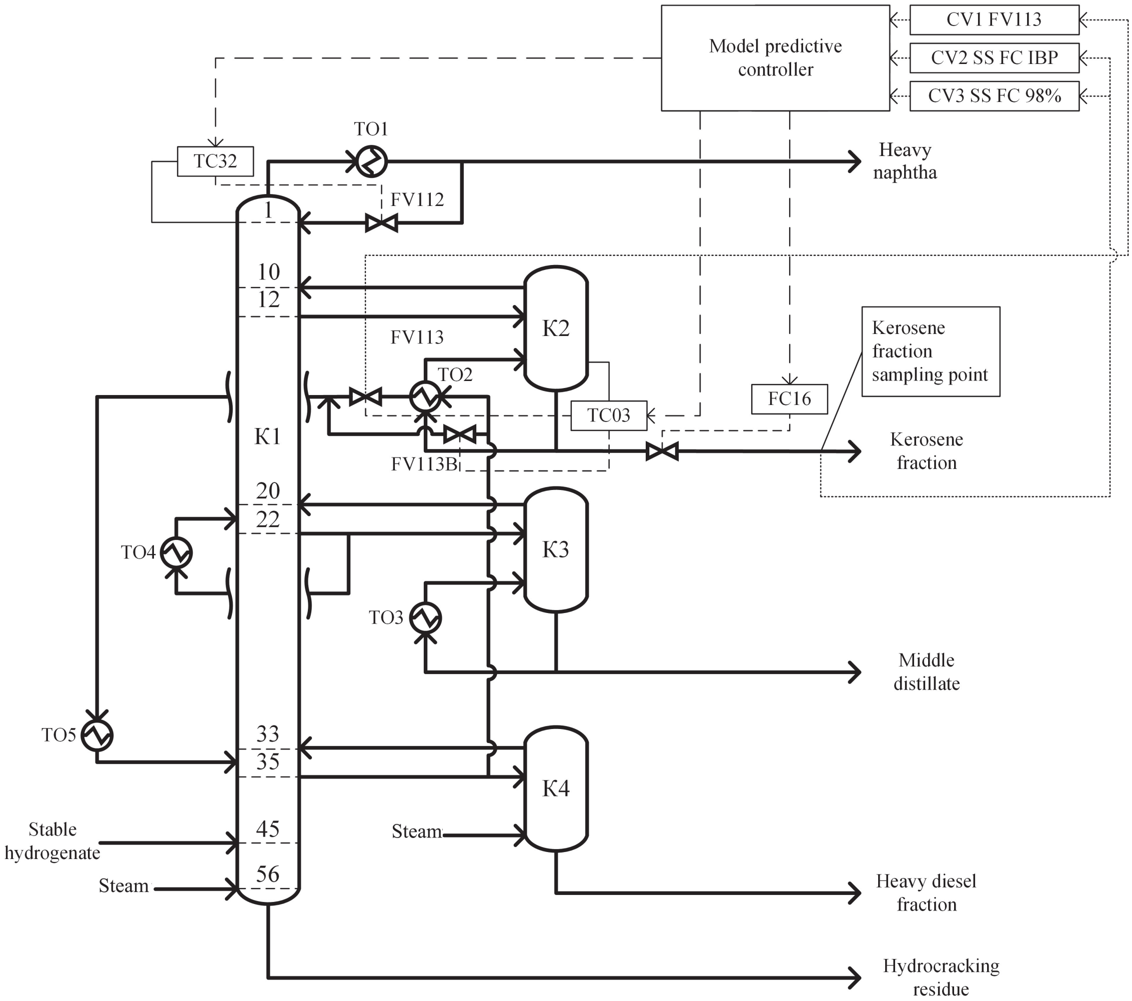

Fig. 1. Diagram of the process unit and control system: K1 – complex distillation column; K2 – kerosene fraction stripping column; K3 – diesel fraction stripping column; K4 – heavy diesel fraction stripping column; TO1 – gasoline fraction vapor condenser; TO2 – K2 column thermosiphon; TO3 – K3 column thermosiphon; TO4 – top pumparound stream cooler; TO5 – bottom pumparound stream cooler; FV112 – valve on the top reflux line of the K1 column; FV113 – valve at the outlet of the TO2 column; FV113B – valve on the TO2 bypass; SS FC IBP – soft sensor (SS) of the initial boiling point of the kerosene fraction; SS FC 98% – SS of the boiling point of the 98% kerosene fraction.

The control system consisting of n_{CV} controlled and n_{MV} control variables is considered. The MPC generates values of increments of control actions, taking into account the predicted values of the output variables by solving the quadratic optimization problem on each time period k:

$$\min_{\Delta \boldsymbol{U}_k} \Phi_k = \frac{1}{2}\Delta \boldsymbol{U}_k^{\mathrm{T}}\left(\boldsymbol{S}_f^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{S}_f + \boldsymbol{R}\right)\Delta \boldsymbol{U}_k + \left(\hat{\boldsymbol{E}}_k^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{S}_f\right)\Delta \boldsymbol{U}_k \tag{1}$$

subject to:

$$\Bigg[\underbrace{\mathbf{I}_{n_{MV}}\,\Big|\ldots\Big|\,\mathbf{I}_{n_{MV}}}_{n_{MV}\times n_{MV}M}\Bigg]^{\mathrm{T}}\Delta \boldsymbol{u}_k^{LL} \leqslant \Delta \boldsymbol{U}_k \leqslant \Bigg[\underbrace{\mathbf{I}_{n_{MV}}\,\Big|\ldots\Big|\,\mathbf{I}_{n_{MV}}}_{n_{MV}\times n_{MV}M}\Bigg]^{\mathrm{T}}\Delta \boldsymbol{u}_k^{HL}, \tag{2}$$

$$\Bigg[\underbrace{\mathbf{I}_{n_{MV}}\,\Big|\ldots\Big|\,\mathbf{I}_{n_{MV}}}_{n_{MV}\times n_{MV}M}\Bigg]^{\mathrm{T}}\left(\boldsymbol{u}_k^{LL} - \boldsymbol{u}_k\right) \leqslant \mathbf{L}\Delta \boldsymbol{U}_k \leqslant \Bigg[\underbrace{\mathbf{I}_{n_{MV}}\,\Big|\ldots\Big|\,\mathbf{I}_{n_{MV}}}_{n_{MV}\times n_{MV}M}\Bigg]^{\mathrm{T}}\left(\boldsymbol{u}_k^{HL} - \boldsymbol{u}_k\right), \tag{3}$$

where M — control horizon; $\Delta\boldsymbol{U}_k = \left[\ \Delta\boldsymbol{u}_k^{\mathrm{T}} \,\middle|\, \dots \,\middle|\, \Delta\boldsymbol{u}_{k+l}^{\mathrm{T}} \,\middle|\, \dots \,\middle|\, \Delta\boldsymbol{u}_{k+M-1}^{\mathrm{T}}\ \right]^{\mathrm{T}}$ — the vector of MV increments on the control horizon M; $\Delta\boldsymbol{u}_{k+l} = (\Delta u_{k+l,1}\ \dots\ \Delta u_{k+l,r}\ \dots\ \Delta u_{k+l,n_{MV}})^{\mathrm{T}}$ — the vector of MV increments for the lth step forward; $\Delta\boldsymbol{u}_k^{LL} = \left(\Delta u_{k,1}^{LL}\ \dots\ \Delta u_{k,r}^{LL}\ \dots\ \Delta u_{k,n_{MV}}^{LL}\right)^{\mathrm{T}}$ and $\Delta\boldsymbol{u}_k^{HL} = \left(\Delta u_{k,1}^{HL}\ \dots\ \Delta u_{k,r}^{HL}\ \dots\ \Delta u_{k,n_{MV}}^{HL}\right)^{\mathrm{T}}$ — the vectors of lower and upper limits on MV increments; $\boldsymbol{u}_k^{LL} = \left(u_{k,1}^{LL}\ \dots\ u_{k,r}^{LL}\ \dots\ u_{k,n_{MV}}^{LL}\right)^{\mathrm{T}}$ and $\boldsymbol{u}_k^{HL} = \left(u_{k,1}^{HL}\ \dots\ u_{k,r}^{HL}\ \dots\ u_{k,n_{MV}}^{HL}\right)^{\mathrm{T}}$ — the vectors of lower and upper limits on MV increments; $\boldsymbol{u}_k$ — the vector of actual MV values at a time k; $\mathbf{L} = \underbrace{\left[\begin{array}{c|c|c|c} \mathbf{I}_{n_{MV}} & 0 & \cdots & 0 \\ \hline \mathbf{I}_{n_{MV}} & \mathbf{I}_{n_{MV}} & \ddots & \vdots \\ \hline \vdots & \vdots & \ddots & 0 \\ \hline \mathbf{I}_{n_{MV}} & \mathbf{I}_{n_{MV}} & \cdots & \mathbf{I}_{n_{MV}} \end{array}\right]}_{n_{MV}M \times n_{MV}M}$ — the lower triangular block matrix consisting of diagonal unit matrices $\mathbf{I}_{n_{MV}}$; $\boldsymbol{S}_f = \left[\begin{array}{c|c|c|c|c} \boldsymbol{s}_1 & 0 & 0 & \cdots & 0 \\ \hline \boldsymbol{s}_2 & \boldsymbol{s}_1 & 0 & \cdots & 0 \\ \hline \vdots & \vdots & \vdots & \ddots & \vdots \\ \hline \boldsymbol{s}_j & \boldsymbol{s}_{j-1} & \boldsymbol{s}_{j-2} & \cdots & \boldsymbol{s}_{j-M+1} \\ \hline \vdots & \vdots & \vdots & \ddots & \vdots \\ \hline \boldsymbol{s}_P & \boldsymbol{s}_{P-1} & \boldsymbol{s}_{P-2} & \cdots & \boldsymbol{s}_{P-M+1} \end{array}\right]$ — the matrix of plant dynamics formed from step response (SR) coefficients; $\boldsymbol{s}_j = \begin{pmatrix} s_{j|1,1} & \cdots & s_{j|1,r} & \cdots & s_{j|1,n_{MV}} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ s_{j|q,1} & \cdots & s_{j|q,r} & \cdots & s_{j|q,n_{MV}} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ s_{j|n_{CV},1} & \cdots & s_{j|n_{CV},r} & \cdots & s_{j|n_{CV},n_{MV}} \end{pmatrix}$ — the matrix of jth SR coefficients for CV and MV pairs; $s_{j|q,r}$ — the jth SR coefficient for qth CV and rth MV; $\boldsymbol{Q} = \operatorname{diag}\left(\left[\underbrace{\mathbf{I}_{n_{CV}} \,\middle|\, \dots \,\middle|\, \mathbf{I}_{n_{CV}}}_{n_{CV} \times n_{CV}P}\right]^{\mathrm{T}} \boldsymbol{w}_{CV}\right)$ — a matrix for taking into account CV weight coefficients when calculating (1); $\boldsymbol{w}_{CV} = (Q_1\ \dots\ Q_q\ \dots\ Q_{n_{CV}})^{\mathrm{T}}$ — the vector of CV weights; P — the prediction horizon; $\boldsymbol{R} = \operatorname{diag}\left(\left[\underbrace{\mathbf{I}_{n_{MV}} \,\middle|\, \dots \,\middle|\, \mathbf{I}_{n_{MV}}}_{n_{MV} \times n_{MV}M}\right]^{\mathrm{T}} \boldsymbol{w}_{MV}\right)$ — a matrix for taking into account MV weight coefficients when calculating (1); $\boldsymbol{w}_{MV} = (R_1\ \dots\ R_r\ \dots\ R_{n_{MV}})^{\mathrm{T}}$ — the vector of MV weights; $\hat{\boldsymbol{E}}_k = \left[\ \boldsymbol{e}_{k|1}^{\mathrm{T}} \,\middle|\, \dots \,\middle|\, \boldsymbol{e}_{k|j}^{\mathrm{T}} \,\middle|\, \dots \,\middle|\, \boldsymbol{e}_{k|P}^{\mathrm{T}}\ \right]^{\mathrm{T}}$ — a vector of errors of the prediction of unforced dynamics of all CV from references to the prediction horizon P in time k; $\boldsymbol{e}_{k|j} = \left(e_{k|1,j}\ \dots\ e_{k|q,j}\ \dots\ e_{k|n_{CV},j}\right)^{\mathrm{T}}$ — a vector of errors of the prediction of unforced dynamics of all CV to the jth step forward in time k; $e_{k|q,j} = \begin{cases} y_q^{HL} - \tilde{y}_{k|q,j}, & \tilde{y}_{k|q,j} > y_q^{HL} \\ 0, & y_q^{LL} \leqslant \tilde{y}_{k|q,j} \leqslant y_q^{HL} \\ y_q^{LL} - \tilde{y}_{k|q,j}, & \tilde{y}_{k|q,j} < y_q^{LL} \end{cases}$ — a error of the prediction of unforced dynamics of the qth CV from the reference for the jth step forward in time k; y_q^{LL} — the lower limit of the qth CV; y_q^{HL} — the upper limit of the qth CV. Constraints (2) and (3) reflect the given limits for MV increments and limits for MV values.

3. STATEMENT OF THE PROBLEM

The vector of the optimized parameters $\boldsymbol{W}$ of the controller is as follows:

$$\boldsymbol{W} = \left(\boldsymbol{w}_{CV}^{\mathrm{T}} \,\middle|\, \boldsymbol{w}_{MV}^{\mathrm{T}} \right)^{\mathrm{T}} = \left(Q_1 \ \dots \ Q_q \ \dots \ Q_{n_{CV}} \ \ R_1 \ \dots \ R_r \ \dots \ R_{n_{MV}}\right)^{\mathrm{T}}.$$

In the industrial conditions, the models used in the controller may not correspond to the plant; in this regard, it is assumed that the values of the parameters of the transfer matrix of the plant model are in a known predetermined range. Therefore, in order to ensure the robustness of the MPC, it is necessary to determine the weights $\boldsymbol{W}$, taking into account the change in the parameters of the plant in the permissible range. Since the number of possible scenarios of variation of parameters in the range is large, in order to reduce the computational load, a limited number of scenarios N_S are used that reflect the maximum possible deviations of the parameters of the models used in the controller from the plant.

Determination of tuning parameters $\hat{\boldsymbol{W}}$ is carried out by solving the optimization problem with a given control performance function J:

$$\hat{\boldsymbol{W}} = \underset{\boldsymbol{W}\in\mathbb{R}_{>0}}{\arg\min} J\left(\boldsymbol{W}\right).$$

The following criterion is proposed as a control performance metric:

$$J = \sum_{m=1}^{N_S} \sum_{q=1}^{n_{CV}} \left(\sum_{k=1}^{N_T} D_{q,k}^{(m)} \sum_{k=1}^{N_T} B_{q,k}^{(m)} \right), \tag{4}$$

where $D_{q,k}^{(m)} = \begin{cases} 0, & y_q^{LL} \leqslant y_{q,k}^{(m)}\left(\boldsymbol{W}\right) \leqslant y_q^{HL} \\ \left| y_q^{LL} - y_{q,k}^{(m)}\left(\boldsymbol{W}\right) \right|, & y_{q,k}^{(m)}\left(\boldsymbol{W}\right) < y_q^{LL} \\ \left| y_{q,k}^{(m)}\left(\boldsymbol{W}\right) - y_q^{HL} \right|, & y_{q,k}^{(m)}\left(\boldsymbol{W}\right) > y_q^{HL} \end{cases}$ — the value of exceeding the permissible boundaries of the qth CV at a time k; N_T — the number of simulation time cycles; $y_{q,k}^{(m)}\left(\boldsymbol{W}\right)$ — the value of the qth CV at a time k, obtained when used a controller with parameters $\boldsymbol{W}$ for scenario m; $B_{q,k}^{(m)} = \begin{cases} 0, & y_q^{LL} \leqslant y_{q,k}^{(m)}\left(\boldsymbol{W}\right) \leqslant y_q^{HL} \\ 1, & y_{q,k}^{(m)}\left(\boldsymbol{W}\right) < y_q^{LL} \ \vee \ y_{q,k}^{(m)}\left(\boldsymbol{W}\right) > y_q^{HL} \end{cases}$ — the flag value (discrete value) of violating the boundaries of the qth CV at a time k obtained for a controller with parameters $\boldsymbol{W}$ for scenario m.

The proposed criterion (4) takes into account the duration (time) and absolute values of the output of CVs beyond the boundaries. Uncertainty consideration is implemented by summarizing the values $\sum_{q=1}^{n_{CV}} \left(\sum_{k=1}^{N_T} D_{q,k}^{(m)} \sum_{k=1}^{N_T} B_{q,k}^{(m)} \right)$ for each of the N_S selected scenarios of parameters of controller model.

4. THE PROPOSED ALGORITHM FOR OBTAINING THE OPTIMAL PARAMETERS OF THE MPC

Due to the discontinuity of the proposed criterion (4) the problem of finding the optimal parameters of the controller cannot be solved by quadratic optimization methods. For this reason, the proposed algorithm for determining the optimal parameters of the controller is based on the grid search method [18, 19], which assumes that the value of the control performance function is calculated for all combinations of the values of the optimized variables within the given search space and the combination with the lowest value of the criterion is selected. The proposed algorithm is presented in Table 1.

Table 1. Algorithm for determining the optimal parameters of the MPC

Inputs: $\boldsymbol{W}_{vars}$, $\boldsymbol{\Phi}$, J^*	
Outputs: $\hat{\boldsymbol{W}}$	
1.	For $v = 1 \ldots N_W$:
2.	Set $\tilde{J} = 0$
3.	For $m = 1 \ldots N_S$:
4.	Reading $\boldsymbol{Y}^{m,v}$ if using $[\boldsymbol{\Phi}]_m$
5.	Calculating $\tilde{J} := \tilde{J} + \sum_{q=1}^{n_{CV}} \left(\sum_{k=1}^{N_T} D_{q,k}^{(m)} \sum_{k=1}^{N_T} B_{q,k}^{(m)} \right)$
6.	If $\tilde{J} < J^*$:
7.	$\hat{\boldsymbol{W}} := [\boldsymbol{W}_{vars}]_v$
8.	$J^* := \tilde{J}$
9.	Return $\hat{\boldsymbol{W}}$

For the algorithm to work, it is necessary to set the matrix of combinations of possible values of the controller parameters $\boldsymbol{W}_{vars}$:

$$\boldsymbol{W}_{vars} = \begin{bmatrix} Q_{1,1} & \ldots & Q_{1,q} & \ldots & Q_{1,n_{CV}} & R_{1,1} & \ldots & R_{1,r} & \ldots & R_{1,n_{MV}} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Q_{v,1} & \ldots & Q_{v,q} & \ldots & Q_{v,n_{CV}} & R_{v,1} & \ldots & R_{v,r} & \ldots & R_{v,n_{MV}} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Q_{N_W,1} & \ldots & Q_{N_W,q} & \ldots & Q_{N_W,n_{CV}} & R_{N_W,1} & \ldots & R_{N_W,r} & \ldots & R_{N_W,n_{MV}} \end{bmatrix},$$

where N_W — the number of combinations of the parameters of the controller.

In this paper, first-order transfer function with time-delay is used as the transfer functions of the model of the plant $F(s) = \frac{g}{(\tau s+1)} e^{-\theta s}$ due to its widespread in practice. To take into account the uncertainty in the algorithm for finding the optimal parameters of the controller, it is necessary to set the matrix of scenarios for the parameters of the transfer functions used:

$$\boldsymbol{\Phi} = \left[\begin{array}{c|c|c} \boldsymbol{G}_1 & \boldsymbol{T}_1 & \boldsymbol{\Theta}_1 \\ \hline \vdots & \vdots & \vdots \\ \hline \boldsymbol{G}_m & \boldsymbol{T}_m & \boldsymbol{\Theta}_m \\ \hline \vdots & \vdots & \vdots \\ \hline \boldsymbol{G}_{N_S} & \boldsymbol{T}_{N_S} & \boldsymbol{\Theta}_{N_S} \end{array} \right],$$

where $\boldsymbol{G}_m = \left(g_{i,j}^{(m)} \right)_{1 \leqslant i \leqslant n_{CV}, 1 \leqslant j \leqslant n_{MV}}$ — the matrix of the controller's model gain coefficients for the scenario m; $\boldsymbol{T}_m = \left(\tau_{i,j}^{(m)} \right)_{1 \leqslant i \leqslant n_{CV}, 1 \leqslant j \leqslant n_{MV}}$ — the matrix of the controller's model time constants for the scenario m; $\boldsymbol{\Theta}_m = \left(\theta_{i,j}^{(m)} \right)_{1 \leqslant i \leqslant n_{CV}, 1 \leqslant j \leqslant n_{MV}}$ — the matrix of the controller's model delay values for the scenario m.

During the operation of the algorithm, for each scenario of the parameters of the transfer functions and for each combination of parameters of the controllers, the values of the measured output vector are read for the entire simulation time:

$$\boldsymbol{Y}^{(m,v)} = \left[\, \boldsymbol{y}_1^{(m)} \left([\boldsymbol{W}_{vars}]_v \right) \Big| \ldots \Big| \, \boldsymbol{y}_k^{(m)} \left([\boldsymbol{W}_{vars}]_v \right) \Big| \ldots \Big| \, \boldsymbol{y}_{N_T}^{(m)} \left([\boldsymbol{W}_{vars}]_v \right) \right],$$

where $\boldsymbol{y}_k^{(m)}([\boldsymbol{W}_{vars}]_v) = \left(y_{1,k}^{(m)}([\boldsymbol{W}_{vars}]_v) \ \dots \ y_{q,k}^{(m)}([\boldsymbol{W}_{vars}]_v) \ \dots \ y_{n_{CV},k}^{(m)}([\boldsymbol{W}_{vars}]_v)\right)^{\mathrm{T}}$ — a vector of CV values at time k when using the scenario m of transfer functions parameters and a combination v of controller parameters; $[\cdot]_v$ — the vth row of the matrix.

At the beginning of the algorithm J^*, it is necessary to set some large threshold value of the function so that when using the combination 1 of the controller parameters, the calculated value of the function $\tilde{J}$ is less than J^*.

5. RESULTS AND DISCUSSION

Table 2 shows the transfer functions of the considered model of the plant.

Since the impact of various unmeasurable disturbances (UD) is possible on a real plant, for example, a change in the feed composition or a change in the characteristics of the coolant, additional transfer functions have been introduced into the plant model to take into account possible UD disturbances. These disturbances are used in the search for optimal parameters $\boldsymbol{W}$ in such a way as to create conflict situations in terms of priorities, when it is possible to keep only one CV within the boundaries.

Table 2. Coefficients of the transfer fumctions of the plant model

	MV1 TC32	MV2 TC03	MV3 FC16	UD1	UD2	UD3
CV1 FV113	$g_{1,1} = -6$ $\tau_{1,1} = 70$ $\theta_{1,1} = 30$	$g_{1,2} = 4$ $\tau_{1,2} = 70$ $\theta_{1,2} = 10$	$g_{1,3} = 0.3$ $\tau_{1,3} = 80$ $\theta_{1,3} = 15$	$g_{1,4} = 10$ $\tau_{1,4} = 30$ $\theta_{1,4} = 0$	$g_{1,5} = 0$ $\tau_{1,5} = 0$ $\theta_{1,5} = 0$	$g_{1,6} = 0$ $\tau_{1,6} = 0$ $\theta_{1,6} = 0$
CV2 K_IBP	$g_{2,1} = 2$ $\tau_{2,1} = 40$ $\theta_{2,1} = 20$	$g_{2,2} = 3.5$ $\tau_{2,2} = 30$ $\theta_{2,2} = 10$	$g_{2,3} = 0$ $\tau_{2,3} = 0$ $\theta_{2,3} = 0$	$g_{2,4} = 0$ $\tau_{2,4} = 0$ $\theta_{2,4} = 0$	$g_{2,5} = 10$ $\tau_{2,5} = 30$ $\theta_{2,5} = 0$	$g_{2,6} = 0$ $\tau_{2,6} = 0$ $\theta_{2,6} = 0$
CV3 K_T95	$g_{3,1} = 0.25$ $\tau_{3,1} = 55$ $\theta_{3,1} = 25$	$g_{3,2} = 0$ $\tau_{3,2} = 0$ $\theta_{3,2} = 0$	$g_{3,3} = 0.55$ $\tau_{3,3} = 15$ $\theta_{3,3} = 10$	$g_{3,4} = 0$ $\tau_{3,4} = 0$ $\theta_{3,4} = 0$	$g_{3,5} = 0$ $\tau_{3,5} = 0$ $\theta_{3,5} = 0$	$g_{3,6} = 10$ $\tau_{3,6} = 30$ $\theta_{3,6} = 0$

For the controller UD1, UD2 and UD3 are unmeasurable, therefore it is assumed that the controller matrices for the case without uncertainty coincide with the fragments of the plant model matrices:

$$\hat{\boldsymbol{G}} = (g_{i,j})_{1 \leqslant i,j \leqslant 3}, \quad \hat{\boldsymbol{T}} = (\tau_{i,j})_{1 \leqslant i,j \leqslant 3}, \quad \hat{\boldsymbol{\Theta}} = (\theta_{i,j})_{1 \leqslant i,j \leqslant 3}.$$

In this paper, the uncertainty, considered as a mismatch between the controller and plant parameter matrices, is set intervalized as a parameter difference of 35%. The parameters of the controller matrices with uncertainty are described as follows:

$$\overline{\boldsymbol{G}} = \begin{bmatrix} -8.1 & 5.4 & 0.4 \\ 2.7 & 4.7 & 0 \\ 0.3 & 0 & 0.7 \end{bmatrix} \approx 1.35\hat{\boldsymbol{G}}, \quad \underline{\boldsymbol{G}} = \begin{bmatrix} -3.9 & 2.6 & 0.2 \\ 1.3 & 2.3 & 0 \\ 0.2 & 0 & 0.4 \end{bmatrix} \approx 0.65\hat{\boldsymbol{G}},$$

$$\overline{\boldsymbol{T}} = \begin{bmatrix} 94.5 & 94.5 & 108 \\ 54 & 40.5 & 0 \\ 74.3 & 0 & 20.3 \end{bmatrix} \approx 1.35\hat{\boldsymbol{T}}, \quad \underline{\boldsymbol{T}} = \begin{bmatrix} 45.5 & 45.5 & 52 \\ 26 & 19.5 & 0 \\ 35.8 & 0 & 9.8 \end{bmatrix} \approx 0.65\hat{\boldsymbol{T}},$$

$$\overline{\boldsymbol{\Theta}} = \begin{bmatrix} 40.5 & 13.5 & 20.3 \\ 27 & 13.5 & 0 \\ 33.8 & 0 & 13.5 \end{bmatrix} \approx 1.35\hat{\boldsymbol{\Theta}}, \quad \underline{\boldsymbol{\Theta}} = \begin{bmatrix} 19.5 & 6.5 & 9.8 \\ 13 & 6.5 & 0 \\ 16.3 & 0 & 6.5 \end{bmatrix} \approx 0.65\hat{\boldsymbol{\Theta}}.$$

The given scenarios of the parameters of the controller matrices, considered in the search for optimal parameters $\hat{\boldsymbol{W}}$, are presented in Table 3.

Table 3. Options for model parameters used in the MPC

Index m of the scenario of models used in the controller	1	2	3	4	5	6	7	8	9
Parameters of models used in the controller	$\hat{\boldsymbol{G}}$	$\underline{\boldsymbol{G}}$	$\hat{\boldsymbol{G}}$	$\hat{\boldsymbol{G}}$	$\overline{\boldsymbol{G}}$	$\hat{\boldsymbol{G}}$	$\hat{\boldsymbol{G}}$	$\underline{\boldsymbol{G}}$	$\overline{\boldsymbol{G}}$
	$\hat{\boldsymbol{T}}$	$\hat{\boldsymbol{T}}$	$\underline{\boldsymbol{T}}$	$\hat{\boldsymbol{T}}$	$\hat{\boldsymbol{T}}$	$\overline{\boldsymbol{T}}$	$\hat{\boldsymbol{T}}$	$\underline{\boldsymbol{T}}$	$\overline{\boldsymbol{T}}$
	$\hat{\boldsymbol{\Theta}}$	$\hat{\boldsymbol{\Theta}}$	$\hat{\boldsymbol{\Theta}}$	$\underline{\boldsymbol{\Theta}}$	$\hat{\boldsymbol{\Theta}}$	$\hat{\boldsymbol{\Theta}}$	$\overline{\boldsymbol{\Theta}}$	$\underline{\boldsymbol{\Theta}}$	$\overline{\boldsymbol{\Theta}}$

The plant control simulation time is limited to $N_T = 300$ minutes. The initial values of the plant inputs and outputs are: CV1 = 116, CV2 = 205, CV3 = 75, MV1 = 12, MV2 = 145, MV3 = 241. Restrictions on increments for MV1, MV2 and MV3 are set to ± 0.25, ± 0.25 and ± 0.33, respectively. At the same time, the permissible limits for CV1 = [10 ... 90], CV2 = [135 ... 145], CV3 = [235 ... 245], MV1 = [112,5 ... 118], MV2 = [203 ... 210] and MV3 = [75 ... 85].

The behavior of the specified UDs is defined as follows: from the beginning of the simulation, UD 1, UD 2 and UD 3 are equal to 0, at time 50, the value of UD 1 changed by -1.5, at time 100, the value of UD 2 and UD 3 became equal to -1.7 and 1, respectively, at time 150, the values of UD 2 and UD 3 became equal to 0.

Since the goal is to find the optimal ratios of the weights $\boldsymbol{w}_{CV}$ and $\boldsymbol{w}_{MV}$, the search space is set as follows:

$$\boldsymbol{W}_{vars} \subseteq \mathbb{N}^*,$$

$$\boldsymbol{W}_{vars} = \left\{ (Q_1 Q_2 Q_3 R_1 R_2 R_3)^{\mathrm{T}} \mid Q_1 = 8;\ Q_2, Q_3 \in \{1, \ldots, 7\};\ R_1, R_2, R_3 \in \{1, \ldots, 8\} \right\}.$$

In order to reduce the time needed to iterate through possible controller parameters, the search space is limited to positive integers that are less than or equal to 8, and a fixed weight of the highest priority CV1 is set to 8. In this case, the number of possible combinations of controller parameters is $N_W = 25\,088$. If we increase the search space and limit it to 10, then the number of possible settings will increase to $N_W = 81\,000$, that is, a slight expansion of the search space significantly increases the number of combinations of controller parameters. Due to the fact that the weights of the lower priority CV are always less than the weight of the more important one, the priorities of the control goals described earlier are taken into account. It should be noted that there are no fundamental restrictions preventing the expansion of the search space to the set $\mathbb{R}_{>0}$.

To confirm the efficiency of the proposed criterion, alternative versions of the criterion that consider either only the magnitude or only the timing of CV violations of acceptable limits are considered:

$$J_D = \sum_{m=1}^{N_S} \sum_{q=1}^{n_{CV}} \sum_{k=1}^{N_T} D_{q,k}^{(m)}, \tag{5}$$

$$J_B = \sum_{m=1}^{N_S} \sum_{q=1}^{n_{CV}} \sum_{k=1}^{N_T} B_{q,k}^{(m)}. \tag{6}$$

For a comparative analysis, the optimal MPC parameters were determined for the case without taking into account the uncertainty (only the scenario $m = 1$, Table 3): when using the proposed criterion (4) — $\boldsymbol{W}_{opt}^{(J)} = (8\ \ 2\ \ 5\ \ 1\ \ 1\ \ 7)^{\mathrm{T}}$; by criterion (5) — $\boldsymbol{W}_{opt}^{(J_D)} = (8\ \ 3\ \ 7\ \ 1\ \ 1\ \ 1)^{\mathrm{T}}$; by criterion (6) — $\boldsymbol{W}_{opt}^{(J_B)} = (8\ \ 1\ \ 1\ \ 1\ \ 1\ \ 7)^{\mathrm{T}}$. Figure 2 shows graphs of the change in CV and MV when using the found optimal parameters of the controller $\boldsymbol{W}_{opt}^{(J)}$, $\boldsymbol{W}_{opt}^{(J_D)}$ and $\boldsymbol{W}_{opt}^{(J_B)}$.

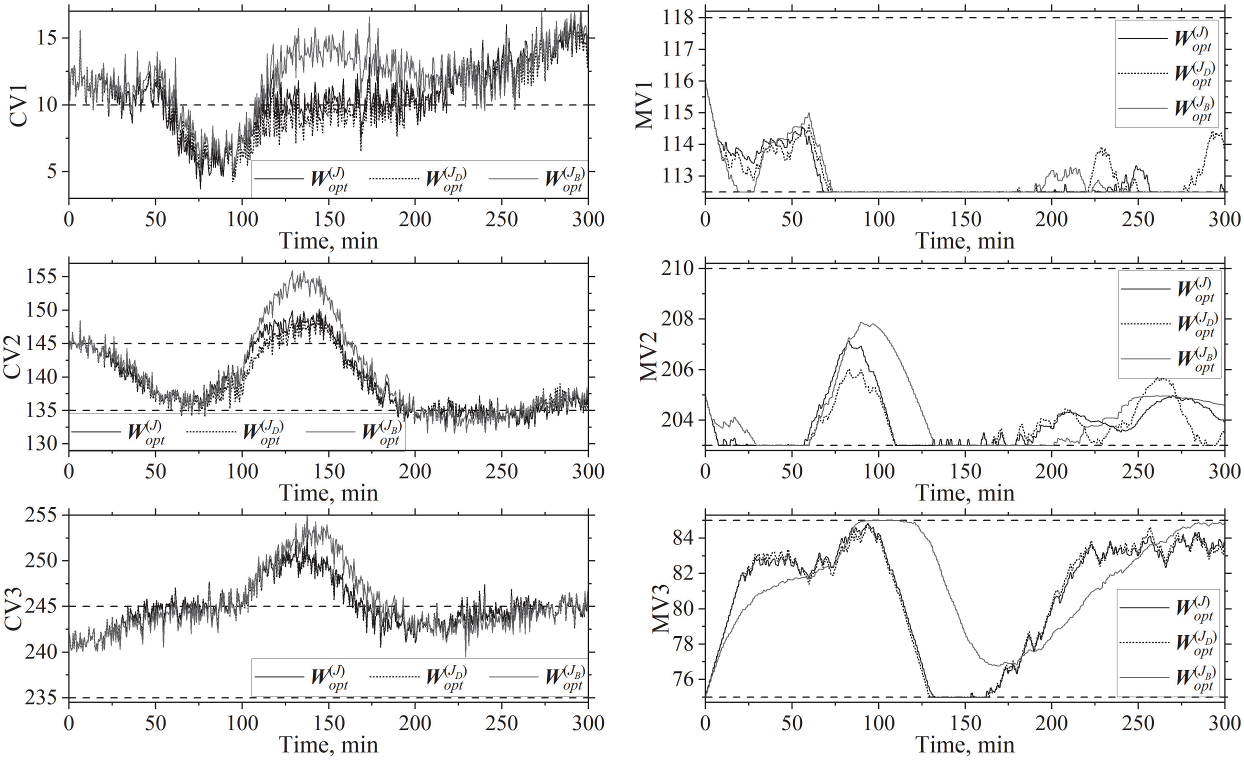

Fig. 2. The simulation results with the MPC parameters $\boldsymbol{W}_{opt}^{(J)}$, $\boldsymbol{W}_{opt}^{(J_D)}$ and $\boldsymbol{W}_{opt}^{(J_B)}$.

According to Fig. 2, it can be concluded that criterion J_B (6) makes it possible to find controller parameters that ensure the fulfillment of priorities, but at the same time the boundaries on CV1 and CV2 are largely violated; criterion J_D (5) makes it possible to find controller parameters that provide a minimum overshooting of boundaries on all CVs, but priorities are not always met; criterion J (4) makes it possible to find such parameters that equally take into account both priorities and values of CV overshooting.

Table 4 shows the optimal parameters (ratios between weights) found by the criterion J for each case of models used in the controller.

Table 4. MPC parameters

$\tilde{m}$ scenario index of controller models	1	2	3	4	5	6	7	8	9
J criterion optimal values of controller parameters for scenario $\tilde{m}$ — $\boldsymbol{W}_{opt}^{(J\|\tilde{m})}$	$\begin{pmatrix}8\\2\\5\\1\\1\\7\end{pmatrix}$	$\begin{pmatrix}8\\3\\1\\8\\1\\7\end{pmatrix}$	$\begin{pmatrix}8\\1\\6\\4\\1\\7\end{pmatrix}$	$\begin{pmatrix}8\\3\\7\\1\\1\\2\end{pmatrix}$	$\begin{pmatrix}8\\1\\7\\7\\1\\7\end{pmatrix}$	$\begin{pmatrix}8\\2\\7\\1\\1\\6\end{pmatrix}$	$\begin{pmatrix}8\\1\\7\\1\\8\\1\end{pmatrix}$	$\begin{pmatrix}8\\3\\1\\1\\1\\7\end{pmatrix}$	$\begin{pmatrix}8\\1\\7\\1\\8\\5\end{pmatrix}$

Taking into account the uncertainty, the optimal parameters were found according to criterion J (4) $\boldsymbol{W}_{opt}^{(J)} = (8\ \ 2\ \ 6\ \ 1\ \ 1\ \ 5)^{\mathrm{T}}$. Figures 3 and 4 show a comparison of the simulation results using the controller parameters $\boldsymbol{W}_{opt}^{(J)}$ and the same weight ratios $\boldsymbol{W}_{std} = (1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1)^{\mathrm{T}}$ for the scenarios $\tilde{m} = 1$ and $\tilde{m} = 8$, accordingly.

The results presented in Figs. 3 and 4 demonstrate the efficiency of the proposed algorithm for finding the optimal parameters of the controller. When using the parameters of the controller $\boldsymbol{W}_{opt}^{(J)}$,

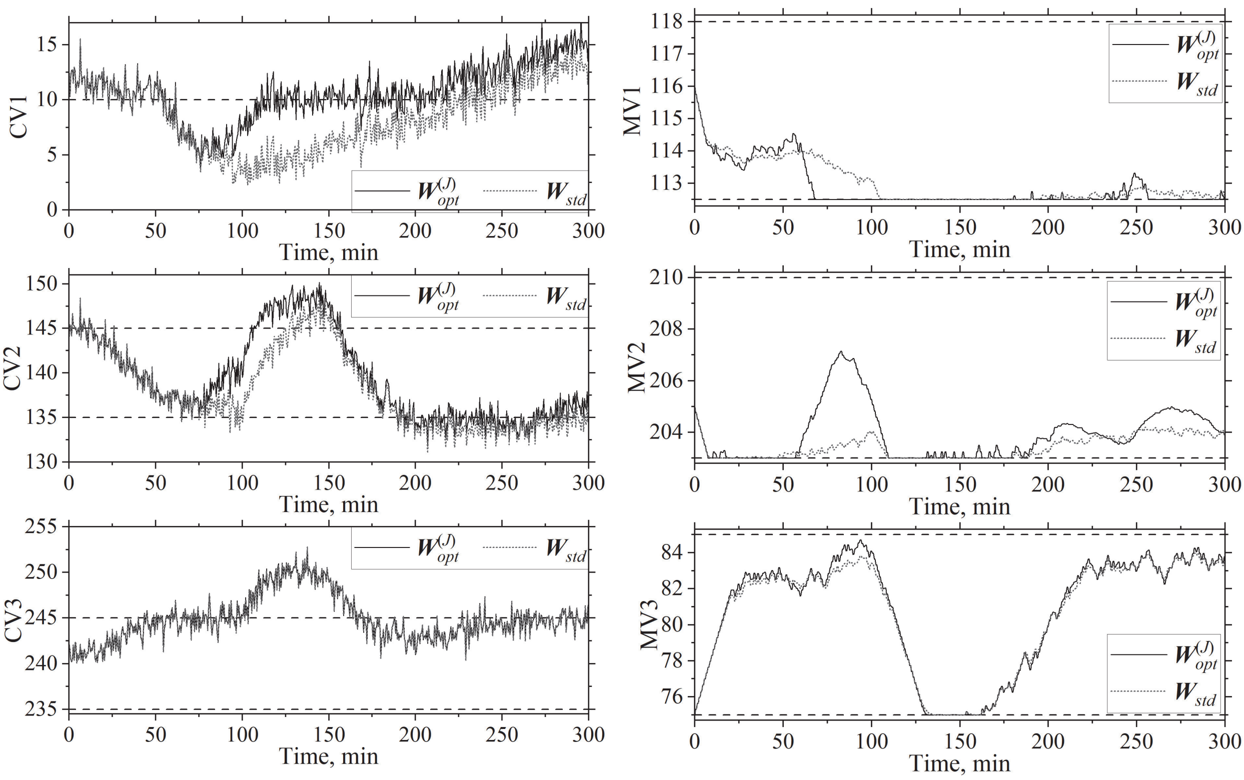

Fig. 3. Comparison of the simulation results using the controller parameters $\boldsymbol{W}_{opt}^{(J)}$ and $\boldsymbol{W}_{std}$ for the scenario $\tilde{m} = 1$.

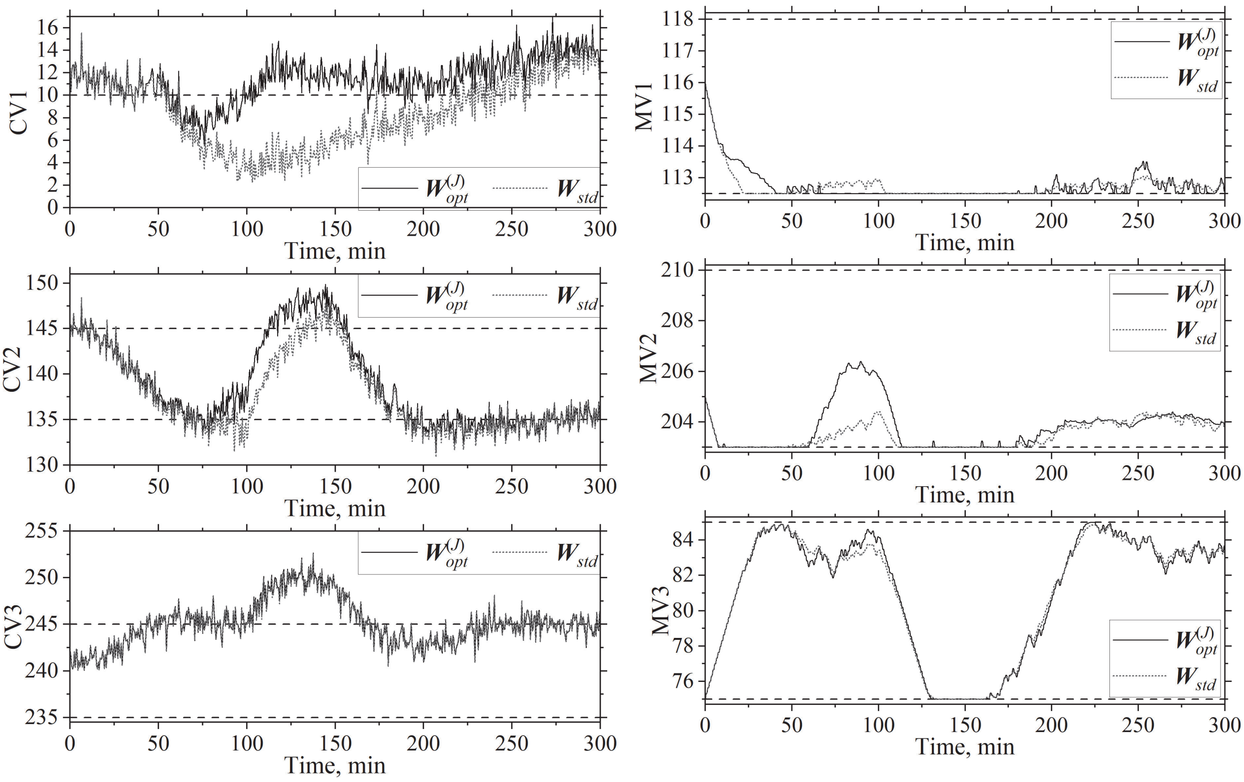

Fig. 4. Comparison of the simulation results using the controller parameters $\boldsymbol{W}_{opt}^{(J)}$ and $\boldsymbol{W}_{std}$ for the scenario $\tilde{m} = 8$.

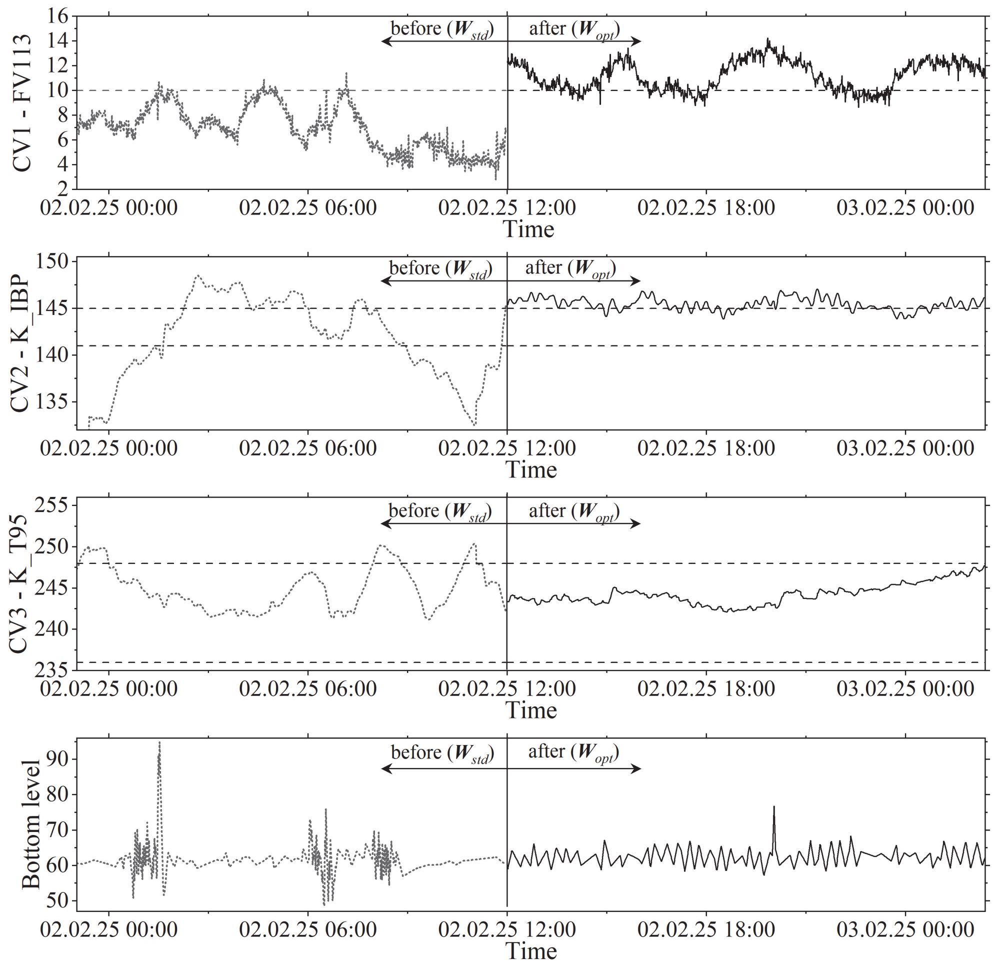

Fig. 5. Comparison of the output variables of the process unit in industrial conditions before and after determining the optimal parameters of the controller.

the time of violation of the boundaries for the most priority CV1 decreased by 42.6% (from 185 min. to 106 min.), 56.2% (from 137 min. to 60 min.) and 29.9% (from 204 min. to 143 min.) for the scenarios of the controller matrices $\tilde{m} = 1$ and $\tilde{m} = 8$, accordingly, in comparison with the use of $\boldsymbol{W}_{std}$. Figure 5 shows the output variables of an industrial process unit when controlled with the parameters $\boldsymbol{W}_{std}$ and $\boldsymbol{W}_{opt}^{(J)}$, respectively. The presented comparison plot makes it possible to conclude that the use of the found optimal parameters of the controller made it possible to prevent a significant violation of the boundaries CV1 — FV113, thereby ensuring a stable accumulation of liquid in the bottom of the K2 column. Also, the optimal parameters found made it possible to stabilize the fractional composition of the kerosene fraction taken from the bottom of the K2 column and to achieve the minimum possible initial boiling point of the kerosene fraction.

6. CONCLUSION

The proposed algorithm for determining the weight matrices enables the determination of the optimal MPC parameters — specifically, the relative weighting factors — under conditions of plant parametric uncertainty, unmeasured disturbances, and control objective prioritization. For the model example under consideration with the controller parameters determined using the proposed algorithm for the highest priority controlled variable, the boundary violation time decreased by an average of 42.9% in comparison with non-optimal controller parameters. Using the example of an industrial MPC of a complex distillation column in a hydrocracking process unit, it demonstrates that the parameters determined by the proposed algorithm enable stabilization of the fractional composition and achievement of the minimum possible initial boiling point of the kerosene fraction withdrawn from the bottom of the stripping column with a thermosiphon, even under low coolant flow conditions.

FUNDING

The research was carried out within the state assignment of IACP FEB RAS on theme FW-2025-0002 (algorithm for obtaining optimal MPC parameters in section 4) and theme FWFW-2021-0003 (implementation of the proposed algorithm and its testing in industrial conditions).

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This paper was recommended for publication by A.A. Galyaev, a member of the Editorial Board